Strangeness S=-3 and -4 baryon-baryon interactions in chiral effective field theory

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Outline

Introduction

- study the role of strangeness in low and medium energy nuclear physics
- test $SU(3)_{flavor}$ symmetry
- H-dibaryon (Jaffe, 1977)
- prerequisite for studies of (Λ, Σ) hypernuclei
- quest for Ξ hypernuclei
  $\longrightarrow$ J-PARC, FAIR
- implications for astrophysics
  $\longrightarrow$ stability of neutron stars
  hyperon stars

YN data [$S = -1$]

- about 35 data points, all from the 1960s
- 10 new data points, from the KEK-PS E251 collaboration (from ≈ 2000)
  (cf. $>$ 4000 NN data for $E_{lab} < 350$ MeV!)

YY data [$S = -2$]

- a few rough estimations of ΞN cross sections from the 1970s
- "more precise" cross sections (for $\Xi^- p$ and $\Xi^- p \to \Lambda\Lambda$) published in 2006
- binding energy of doubly strange hypernuclei (${}^{6}_{\Lambda\Lambda}\mathrm{He}$)

$S = -3, -4$: uncharted territory

YN, YY, ΞY, $\Xi\Xi$ in chiral effective field theory

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three baryon forces and external current operators in a consistent way

Obstacle: YN data base is rather poor
practically no information on YY, ΞY, $\Xi\Xi$
($\rightarrow$ impose $SU(3)_f$ constraints)

few investigations so far (for YN only):

- C.K. Korpa et al., PRC 65 (2001) 015208
- S.R. Beane et al., NPA747 (2005) 55

pion-less theory; Kaplan-Savage-Wise resummation scheme

We follow the scheme of E. Epelbaum et al.

(E. Epelbaum, W. Glöckle, Ulf-G. Meißner, NPA747 (2005) 362)

Power counting

$$V_{\rm eff} \equiv V_{\rm eff}(Q, g, \mu) = \sum_{\nu} (Q/\Lambda)^{\nu}\, \mathcal{V}_{\nu}(Q/\mu, g)$$

- Q ... soft scale (baryon three-momentum, Goldstone boson four-momentum, Goldstone boson mass)
- Λ ... hard scale
- g ... pertinent low–energy constants
- μ ... regularization scale
- $\mathcal{V}_{\nu}$ ... function of order one
- $\nu \geq 0$ ... chiral power

Lowest order (LO): $\nu = 0$

a) non-derivative four-baryon contact terms

b) one-meson (Goldstone boson) exchange diagrams

Leading order (LO) contact term for NN

The LO contact term for the NN interaction:

$$\mathcal{L} = C_i \left(\bar{N}\Gamma_i N\right)\left(\bar{N}\Gamma_i N\right)$$

$$\Gamma_1 = 1\,,\ \Gamma_2 = \gamma^\mu\,,\ \Gamma_3 = \sigma^{\mu\nu}\,,\ \Gamma_4 = \gamma^\mu\gamma_5\,,\ \Gamma_5 = \gamma_5$$

Considering the large components of the nucleon spinors only, the LO contact term becomes

$$\mathcal{L} = -\frac{1}{2}C_S\left(\varphi_N^\dagger\varphi_N\right)\left(\varphi_N^\dagger\varphi_N\right) - \frac{1}{2}C_T\left(\varphi_N^\dagger\boldsymbol{\sigma}\varphi_N\right)\left(\varphi_N^\dagger\boldsymbol{\sigma}\varphi_N\right)$$

The LO contact term potential resulting from the interaction Lagrangian:

$$V^{NN\to NN} = C_S + C_T\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

C_S and C_T ... low-energy constants; to be determined in a fit to the experimental data.

Leading order contact terms for YN and YY

spin-momentum structure of the LO contact term potential resulting from the BB interaction Lagrangian:

$$V^{BB\to BB} = C_S^{BB\to BB} + C_T^{BB\to BB}\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

$SU(3)$ structure for scattering of two octet baryons:
direct product:

$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$

There are only 6 independent low-energy constants for the BB interaction!
(8 independent spin-isospin channels in YN alone!)

C_S^i, C_T^i can be expressed by the coefficients corresponding to the $SU(3)_f$ irreducible representations:
C^1, C^{8_a}, C^{8_s}, C^{10^*}, C^{10}, C^{27}

$SU(3)$ content

BB contact interactions in terms of $SU(3)_f$ irreducible representations

	Channel	Isospin	$V_{^3S_1}$	Isospin	$V_{^1S_0}$
$S = 0$	$NN \to NN$	0	C^{10^*}	1	C^{27}
$S = -1$	$\Lambda N \to \Lambda N$	$\frac{1}{2}$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27} + C^{8_s}\right)$
	$\Lambda N \to \Sigma N$	$\frac{1}{2}$	$\frac{1}{2}\left(-C^{8_a} + C^{10^*}\right)$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27} + C^{8_s}\right)$
	$\Sigma N \to \Sigma N$	$\frac{1}{2}$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27} + 9C^{8_s}\right)$
	$\Sigma N \to \Sigma N$	$\frac{3}{2}$	C^{10}	$\frac{3}{2}$	C^{27}
$S = -3$	$\Xi\Lambda \to \Xi\Lambda$	$\frac{1}{2}$	$\frac{1}{2}\left(C^{8_a} + C^{10}\right)$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27} + C^{8_s}\right)$
	$\Xi\Lambda \to \Xi\Sigma$	$\frac{1}{2}$	$\frac{1}{2}\left(-C^{8_a} + C^{10}\right)$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27} + C^{8_s}\right)$
	$\Xi\Sigma \to \Xi\Sigma$	$\frac{1}{2}$	$\frac{1}{2}\left(C^{8_a} + C^{10}\right)$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27} + 9C^{8_s}\right)$
	$\Xi\Sigma \to \Xi\Sigma$	$\frac{3}{2}$	C^{10^*}	$\frac{3}{2}$	C^{27}
$S = -4$	$\Xi\Xi \to \Xi\Xi$	0	C^{10}	1	C^{27}

10 and 10^* representations interchange their roles when going from the $S = 0, -1$ to the $S = -3, -4$ channels

One pseudoscalar-meson exchange

$$
\begin{aligned}
\mathcal{L} \quad = \quad & -f_{NN\pi}\bar{N}\gamma^\mu\gamma_5\boldsymbol{\tau}N\cdot\partial_\mu\boldsymbol{\pi} + if_{\Sigma\Sigma\pi}\bar{\boldsymbol{\Sigma}}\gamma^\mu\gamma_5\times\boldsymbol{\Sigma}\cdot\partial_\mu\boldsymbol{\pi} \\
& -f_{\Lambda\Sigma\pi}\left[\bar{\Lambda}\gamma^\mu\gamma_5\boldsymbol{\Sigma}+\bar{\boldsymbol{\Sigma}}\gamma^\mu\gamma_5\Lambda\right]\cdot\partial_\mu\boldsymbol{\pi} - f_{\Xi\Xi\pi}\bar{\Xi}\gamma^\mu\gamma_5\boldsymbol{\tau}\Xi\cdot\partial_\mu\boldsymbol{\pi} \\
& -f_{\Lambda NK}\left[\bar{N}\gamma^\mu\gamma_5\Lambda\partial_\mu K + \bar{\Lambda}\gamma^\mu\gamma_5 N\partial_\mu K^\dagger\right] \\
& -f_{\Xi\Lambda K}\left[\bar{\Xi}\gamma^\mu\gamma_5\Lambda\partial_\mu K_c + \bar{\Lambda}\gamma^\mu\gamma_5\Xi\partial_\mu K_c^\dagger\right] \\
& -f_{\Sigma NK}\left[\bar{\boldsymbol{\Sigma}}\cdot\gamma^\mu\gamma_5\partial_\mu K^\dagger\boldsymbol{\tau}N + \bar{N}\gamma^\mu\gamma_5\boldsymbol{\tau}\partial_\mu K\cdot\boldsymbol{\Sigma}\right] \\
& -f_{\Sigma\Xi K}\left[\bar{\boldsymbol{\Sigma}}\cdot\gamma^\mu\gamma_5\partial_\mu K_c^\dagger\boldsymbol{\tau}\Xi + \bar{\Xi}\gamma^\mu\gamma_5\boldsymbol{\tau}\partial_\mu K_c\cdot\boldsymbol{\Sigma}\right] - f_{NN\eta_8}\bar{N}\gamma^\mu\gamma_5 N\partial_\mu\eta \\
& -f_{\Lambda\Lambda\eta_8}\bar{\Lambda}\gamma^\mu\gamma_5\Lambda\partial_\mu\eta - f_{\Sigma\Sigma\eta_8}\bar{\boldsymbol{\Sigma}}\cdot\gamma^\mu\gamma_5\boldsymbol{\Sigma}\partial_\mu\eta - f_{\Xi\Xi\eta_8}\bar{\Xi}\gamma^\mu\gamma_5\Xi\partial_\mu\eta
\end{aligned}
$$

$f_{NN\pi} =$	f	$f_{NN\eta_8} =$	$\frac{1}{\sqrt{3}}(4\alpha-1)f$	$f_{\Lambda NK} =$	$-\frac{1}{\sqrt{3}}(1+2\alpha)f$
$f_{\Xi\Xi\pi} =$	$-(1-2\alpha)f$	$f_{\Xi\Xi\eta_8} =$	$-\frac{1}{\sqrt{3}}(1+2\alpha)f$	$f_{\Xi\Lambda K} =$	$\frac{1}{\sqrt{3}}(4\alpha-1)f$
$f_{\Lambda\Sigma\pi} =$	$\frac{2}{\sqrt{3}}(1-\alpha)f$	$f_{\Sigma\Sigma\eta_8} =$	$\frac{2}{\sqrt{3}}(1-\alpha)f$	$f_{\Sigma NK} =$	$(1-2\alpha)f$
$f_{\Sigma\Sigma\pi} =$	$2\alpha f$	$f_{\Lambda\Lambda\eta_8} =$	$-\frac{2}{\sqrt{3}}(1-\alpha)f$	$f_{\Xi\Sigma K} =$	$-f$

$f = g_A/(2F_\pi)$; $g_A \simeq 1.26$, $F_\pi = 92.4$ MeV

$\alpha = F/(F+D)$ with $g_A = F+D$

One pseudoscalar-meson exchange

$$V^{B_1 B_2 \to B_1' B_2'} = -f_{B_1 B_1' P} f_{B_2 B_2' P} \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k})}{\mathbf{k}^2 + m_P^2}$$

$f_{B_1 B_1' P}$ ... coupling constants
m_P ... mass of the exchanged pseudoscalar meson

- SU(3) breaking due to the mass splitting of the ps mesons (m_π = 138.0 MeV, m_K = 495.7 MeV, m_η = 547.3 MeV) is taken into account

Details:

(H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244)
(H. Polinder, J.H., U.-G. Meißner, PLB 653 (2007) 29)

Coupled channels Lippmann-Schwinger Equation

$$
\begin{aligned}
T_{\rho'\rho}{}^{\nu'\nu,J}(p',p) &= V_{\rho'\rho}{}^{\nu'\nu,J}(p',p) \\
&+ \sum_{\rho'',\nu''} \int_0^\infty \frac{dp''p''^2}{(2\pi)^3} V_{\rho'\rho''}{}^{\nu'\nu'',J}(p',p'') \frac{2\mu_{\nu''}}{p^2 - p''^2 + i\eta} T_{\rho''\rho}{}^{\nu''\nu,J}(p'',p)
\end{aligned}
$$

$$
\begin{aligned}
\rho',\ \rho &= \Lambda N,\ \Sigma N \\
&= \Lambda\Lambda,\ \Sigma\Sigma,\ \Xi N,\ \Sigma\Lambda \\
&= \Xi\Lambda,\ \Xi\Sigma \\
&= \Xi\Xi
\end{aligned}
$$

LS equation is solved for particle channels (in momentum space)

Coulomb interaction is included via the Vincent-Phatak method

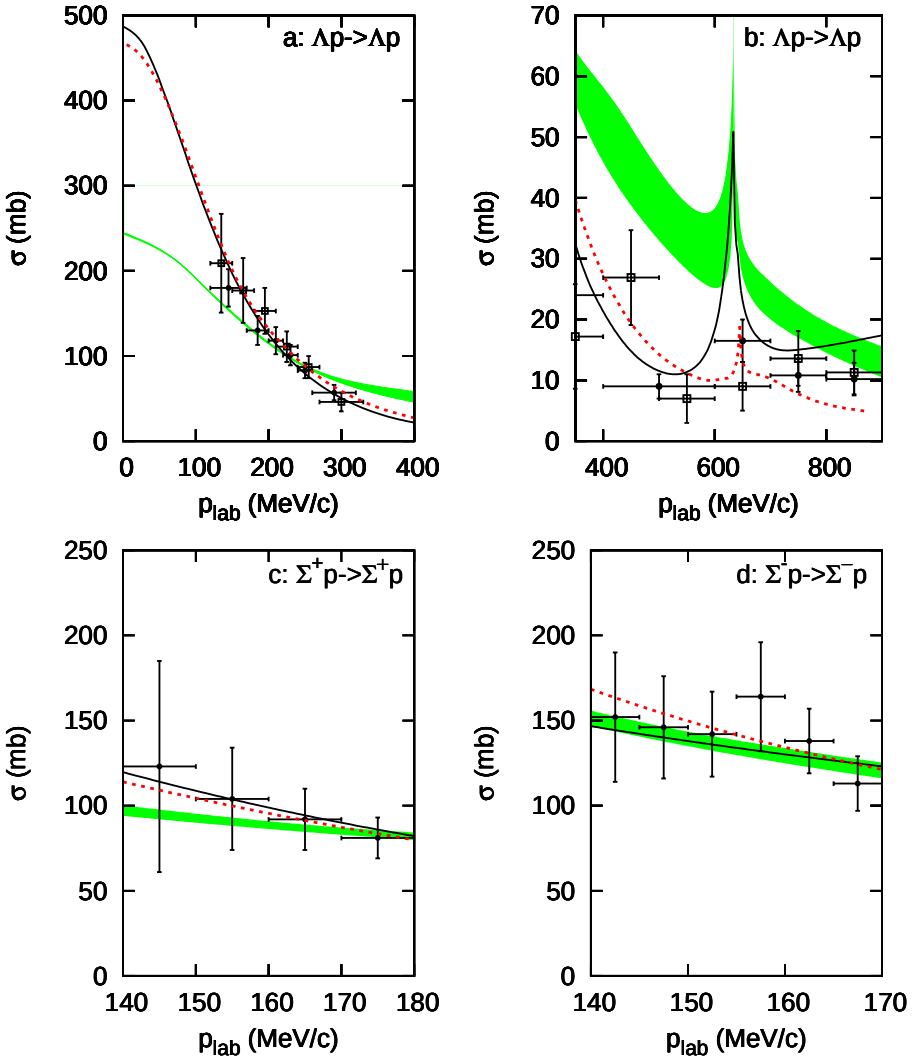
The potential in the LS equation is cut off with the regulator function:

$$
V_{\rho'\rho}{}^{\nu'\nu,J}(p',p) \to f^\Lambda(p') V_{\rho'\rho}{}^{\nu'\nu,J}(p',p) f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^4}
$$

consider values Λ = 550 - 700 MeV

(No $SU(3)$ constraints from the NN sector are imposed!)

YN integrated cross sections

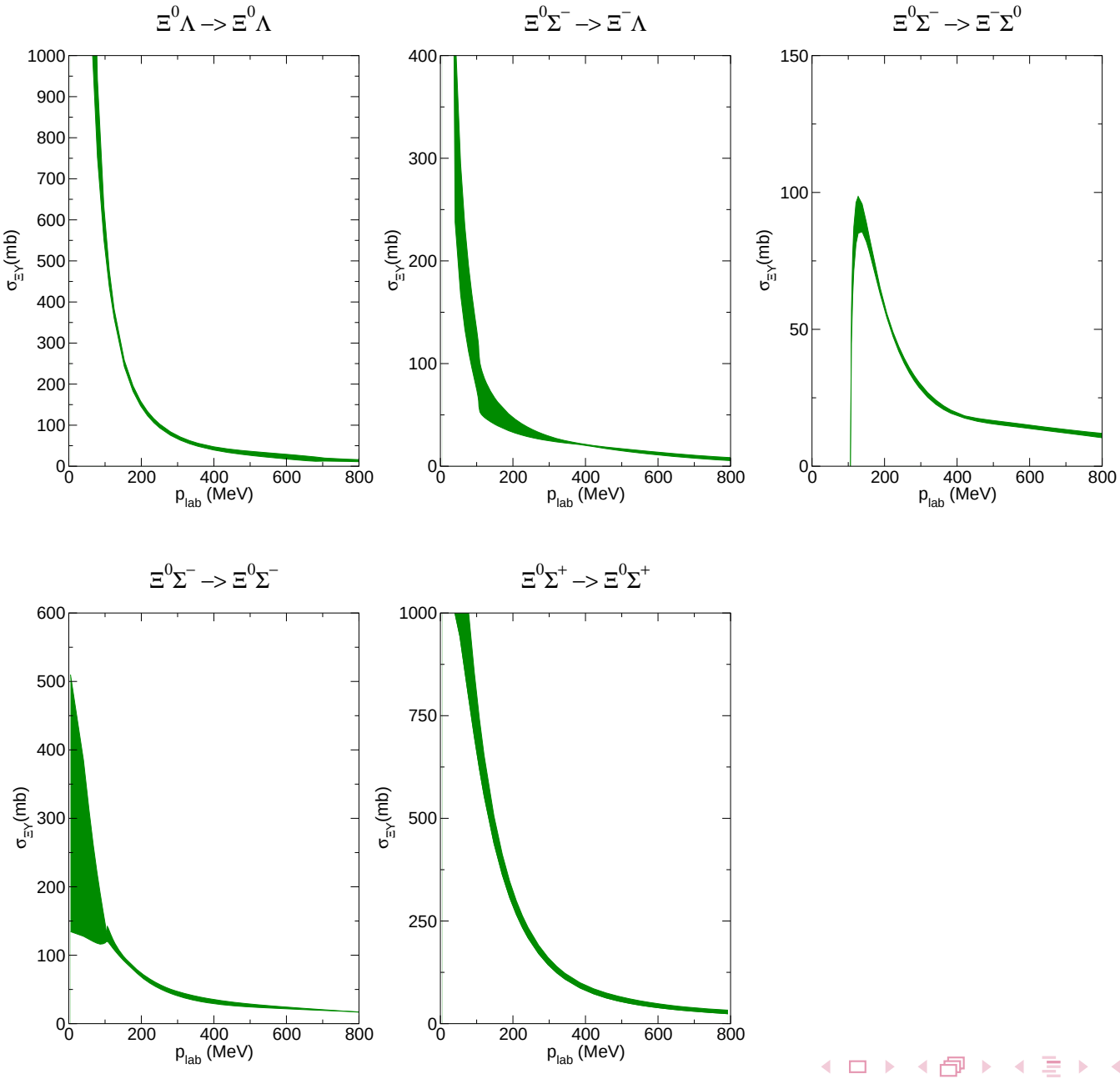


EFT LO
(NPA 779 (2006) 244)

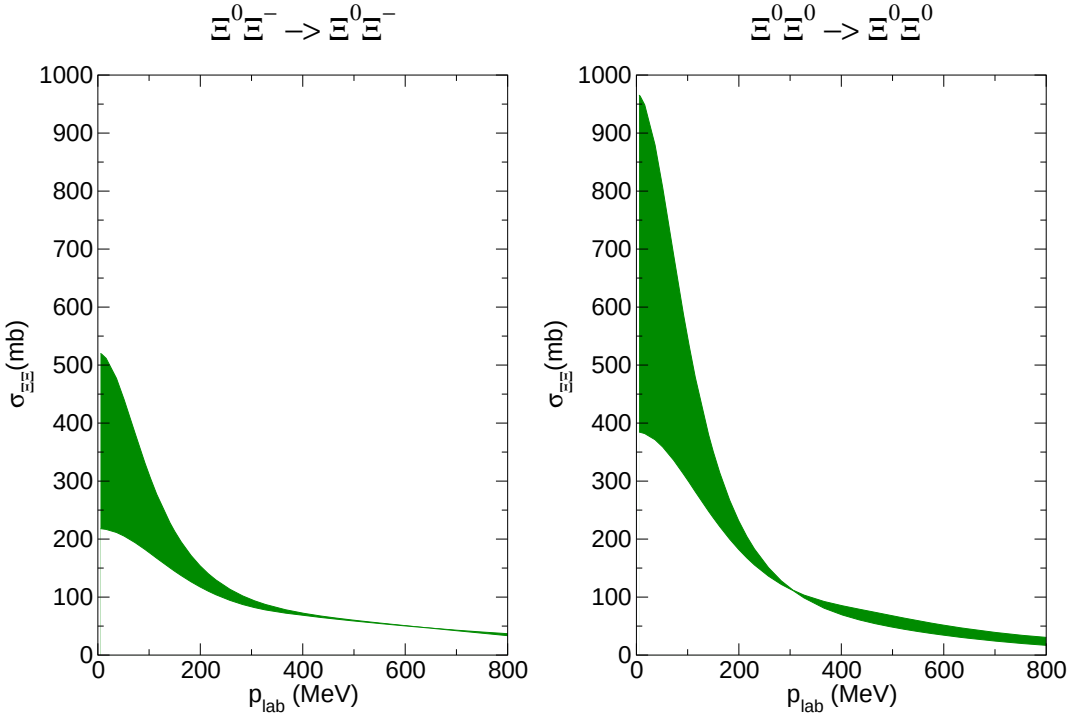
Jülich '04
(PRC 72 (2005) 044005)

Nijmegen NSC97f
(PRC 59 (1999) 21)

Results for $S = -3$ and $S = -4$



ΞΞ integrated cross sections



J.H., U.-G. Meißner, Phys. Lett. B 684 (2010) 275

ΞY and $\Xi\Xi$ scattering lengths [fm]

	EFT LO				NSC97a	NSC97f	fss2
Λ [MeV]	550	600	650	700			
$a_s^{\Xi\Lambda}$	−33.5	35.4	12.7	9.07	−0.80	−2.11	−1.08
$a_t^{\Xi\Lambda}$	0.33	0.33	0.32	0.31	0.54	0.33	0.26
$a_s^{\Xi^0\Sigma^+}$	4.28	3.45	2.97	2.74	4.13	2.32	−4.63
$a_t^{\Xi^0\Sigma^+}$	−2.45	−3.11	−3.57	−3.89	3.21	1.71	−3.48
$a_s^{\Xi\Xi}$	3.92	3.16	2.71	2.47	17.81	2.38	−1.43
$a_t^{\Xi\Xi}$	0.63	0.59	0.55	0.52	0.40	0.48	3.20

(Nijmegen: Stoks & Rijken, PRC 59 (1999) 3009)
(fss2: Fujiwara, Suzuki & Nakamoto, Prog. Part. Nucl. Phys. 58 (2007) 439)

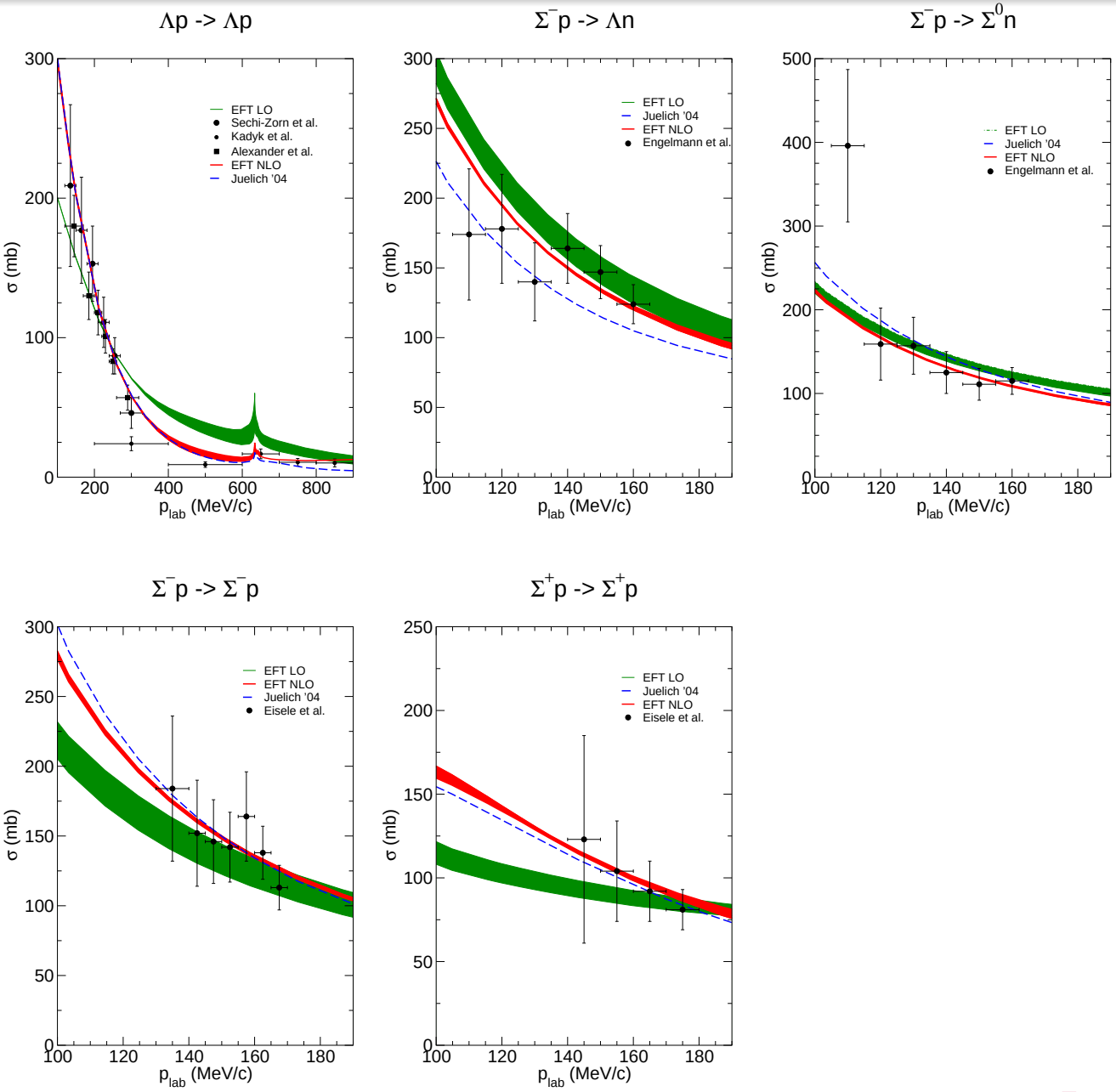
Preliminary (incomplete) NLO results

- Additional (8) contact terms in S-waves are taken into account
- (10) Contact terms in P-waves are not yet included
- Two-pseudoscalar-meson exchange diagrams are missing

- no SU(3) constraints from the NN sector are imposed (SU(3) symmetry is used to relate ΛN and ΣN!)
- leading order SU(3) breaking in the one-boson exchange diagrams (coupling constants) is ignored

$\Rightarrow$ J.H., Nucl. Phys. A 835 (2010) 168

YN integrated cross sections (preliminary)



YN scattering lengths [fm] (preliminary)

	EFT NLO				EFT LO	NSC97f	experiment
Λ [MeV]	550	600	650	700	550		
$a_s^{\Lambda p}$	-2.61	-2.61	-2.59	-2.63	-1.90	-2.51	$-1.8^{+2.3}_{-4.2}$
$a_t^{\Lambda p}$	-1.64	1.63	-1.62	-1.63	-1.22	-1.75	$-1.6^{+1.1}_{-0.8}$
$a_s^{\Sigma^+ p}$	-4.13	-4.11	-3.99	-3.97	-2.24	-4.35	
$a_t^{\Sigma^+ p}$	-0.01	0.01	0.01	0.01	0.70	-0.25	
χ^2	16.8	16.7	16.5	16.9	29.6	16.7	
$({}^3_\Lambda\mathrm{H})$ E_B	-2.34	-2.34	-2.39	-2.38	-2.35	-2.30	$-2.354(50)$

Summary

YN, YY, ΞY, $\Xi\Xi$ interactions based on *EFT*

- approach is based on the Weinberg power counting, analogous to the NN case
- LO potential (contact terms, one-pseudoscalar-meson exchange) is derived imposing $SU(3)_f$ constraints
- Good description of the empirical YN data was achieved (with only 5 free parameters!)
- $\Rightarrow$ Predictions for the $S = -3$ ($\Lambda\Xi$, $\Sigma\Xi$) and $S = -4$ ($\Xi\Xi$) sectors can be made
- Preliminary (incomplete) YN results in next-to-leading order (NLO) look very promising

Outlook

Next tasks:

- A combined study of the NN and YN systems in chiral EFT, based on a complete NLO calculation
- A more thorough exploration of the interrelation between the elementary YN interaction and the properties of light hypernuclei
  - calculate the $YNNN$ bound states
  - consider YNN three-body forces